

# Advanced Computing for Materials and Manufacturing

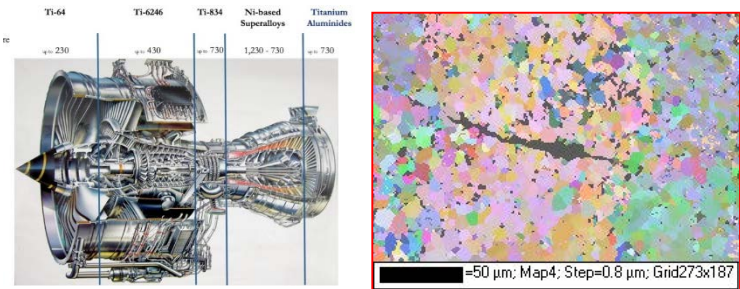
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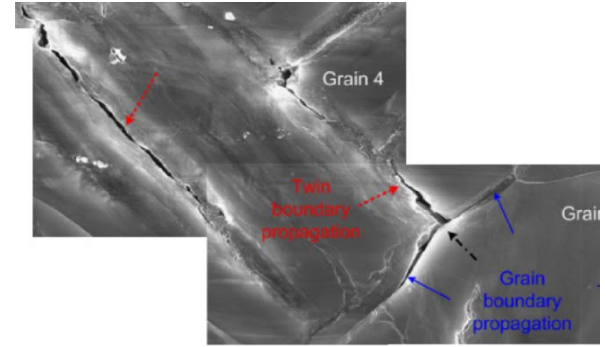
**Workshop on Core Knowledge and Skills for  
Effective Use of Advanced Computation and Data  
Phoenix Convention Center, Phoenix, AZ  
March 11, 2018**

# Challenges in Modeling Materials & Manufacturing: Examples

## Fatigue in Titanium Alloys



## Twinning in Mg Alloys




## Structure-Material Problems in Additive Manufacturing

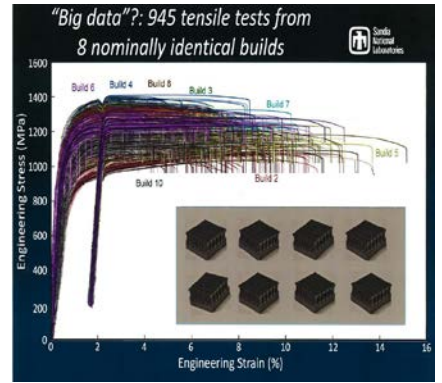
### Topology Optimization



### LENS, DMD



### Microstructure

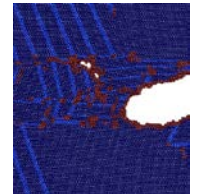
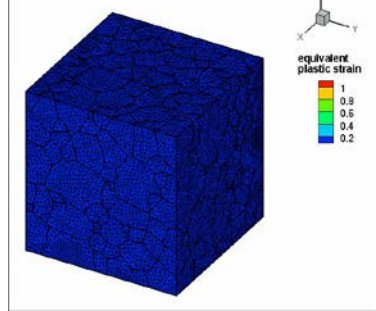
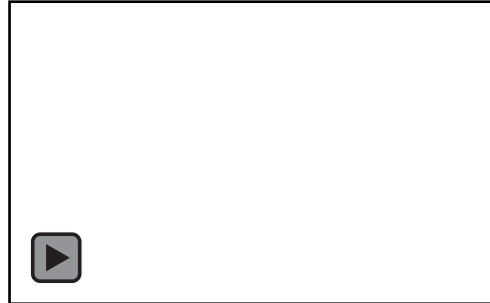
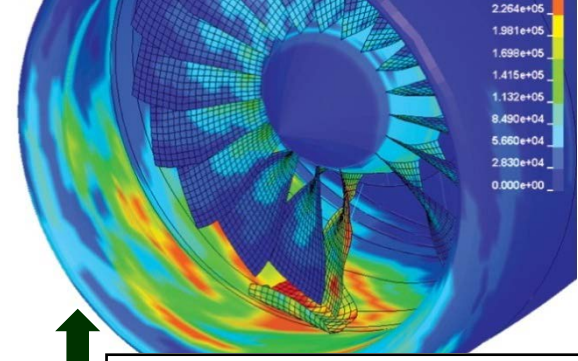
# Material-Structure Modeling

## ICME & ICSME

Integrated Computational Structure-  
Materials Engineering (ICSME) provides  
a Multi-Scale Framework for Structural  
Performance and Life  
*(Location-specific Material Design)*



The ICME framework provides a  
systematic approach for effective tools  
coupling Computational and  
Experimental Materials with  
Computational Mechanics for predicting  
material behavior and life at lower scales.



# Challenges in Computational Modeling at Multiple Scales

- Accurate Representation of Computational Domains at Each Scale for Modeling and Simulations of Relevant Properties and Response Functions: *RVE, SERVE, Domains and Boundary Conditions*
- Methods of Analysis and Simulation at Relevant Scales: *Accuracy and Efficiency*
- Accelerated Time Integration Methods and Multi-time Scaling
- Computational Modeling Across Scales: *Methods of Hierarchical and Concurrent Multiscaling*



# Statistically Equivalent Representative Volume Element (SERVE)

**Microstructural domain, for which statistical distribution functions of morphological parameters or material response functions converge to those for the entire microstructure.**

*Ghosh, Swaminathan I&II (2005), McDowell, Ghosh, Kalidindi (2011), Sundararaghavan, Zabaras (2005), Kumar, Nguyen, DeGraef, Sundararaghavan (2016), Rollett, Robert, Saylor (2006), Guo, Chawla, Jing, Torquato, Jiao (2014)*

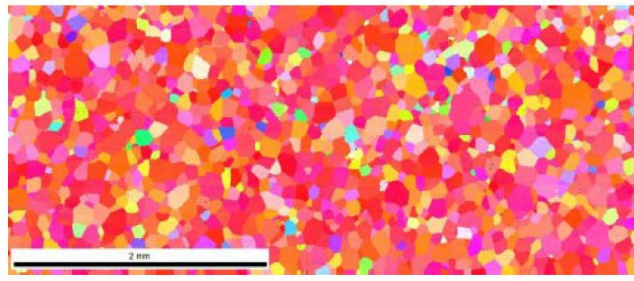
1. Microstructure-based SERVE or M-SERVE, in which morphological and crystallographic characteristics of the microstructure are the sole determinants of the SERVE domain
2. Property-based SERVE or M-SERVE, in which specific properties and response functions are determinants of the SERVE domain

*Groeber, Ghosh, Uchic, Dimiduk I&II (2008), Niezgodna, Turner, Fullwood, Kalidindi (2010)*

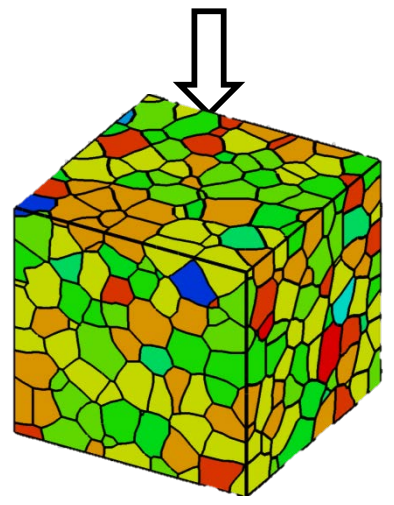
*Bagri, Weber, Lenthe, Pollock, Woodward, Ghosh (2018), Pinz, Weber, Lenthe, Pollock, Uchic, Ghosh (2018)*

# M-SERVE for Ti Alloys

## DREAM.3D and Beyond

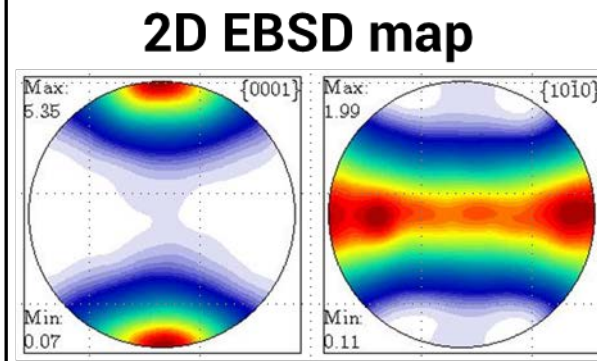
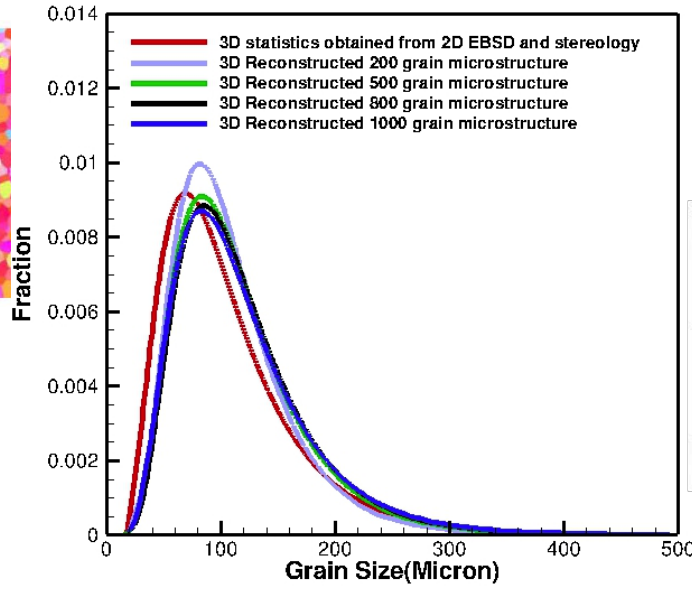


Annealed-rolled Ti-7Al EBSD scan  
(Courtesy of Adam Pilchak, AFRL)

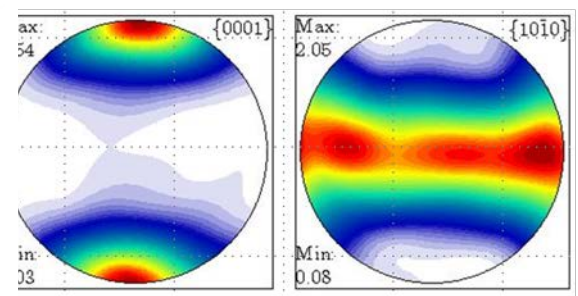
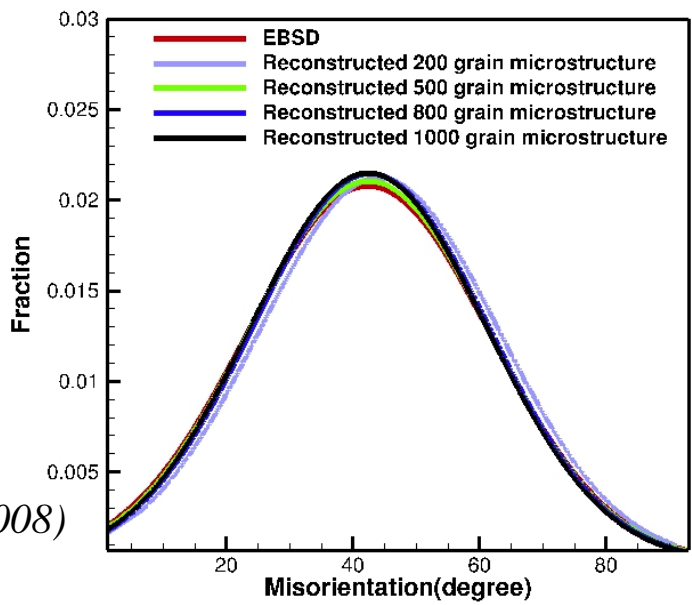


**DREAM.3D**

Groeber, Ghosh, Uchic, Dimiduk (2008)  
Groeber, Jackson (2014)

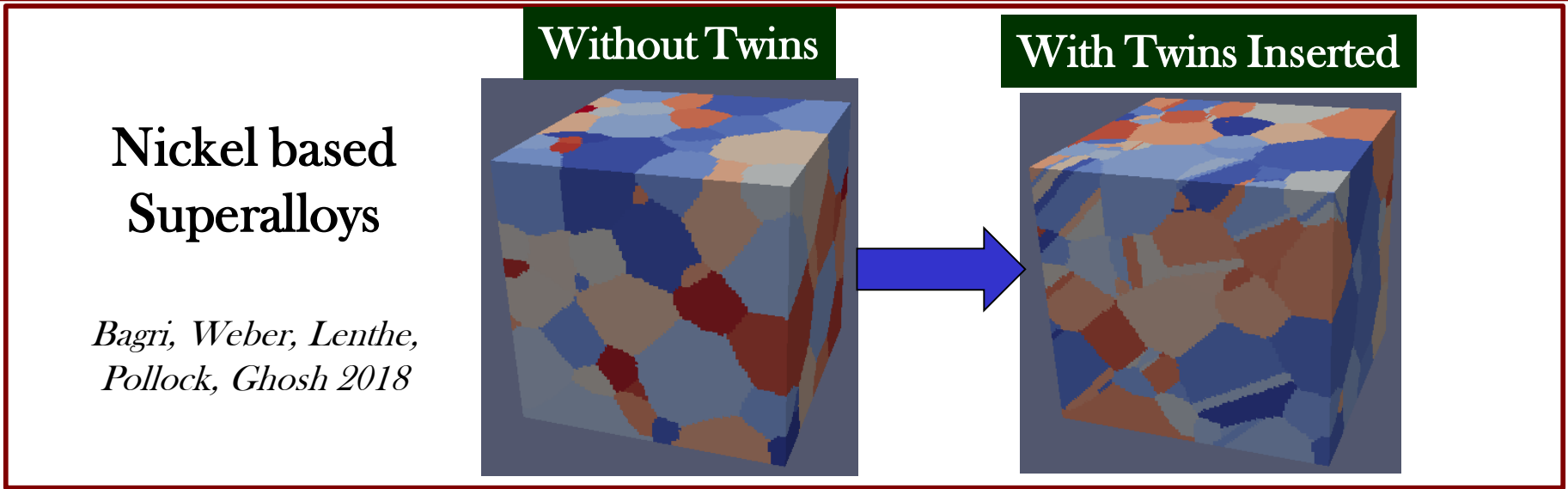
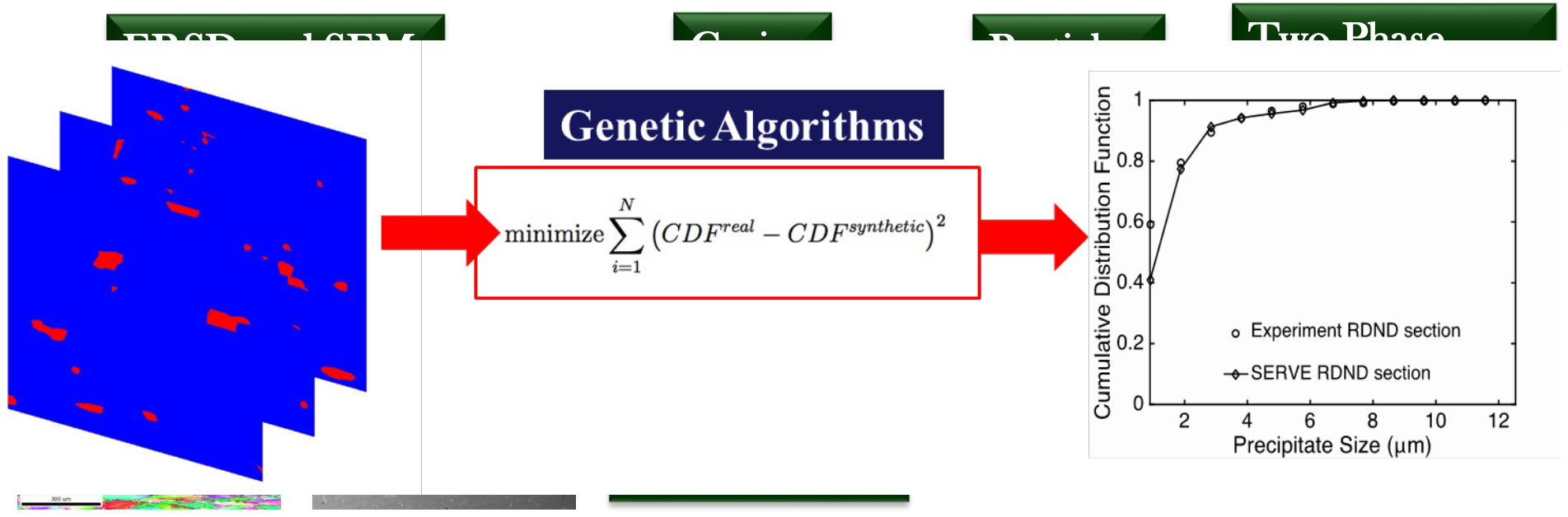


**500-grain micro.**



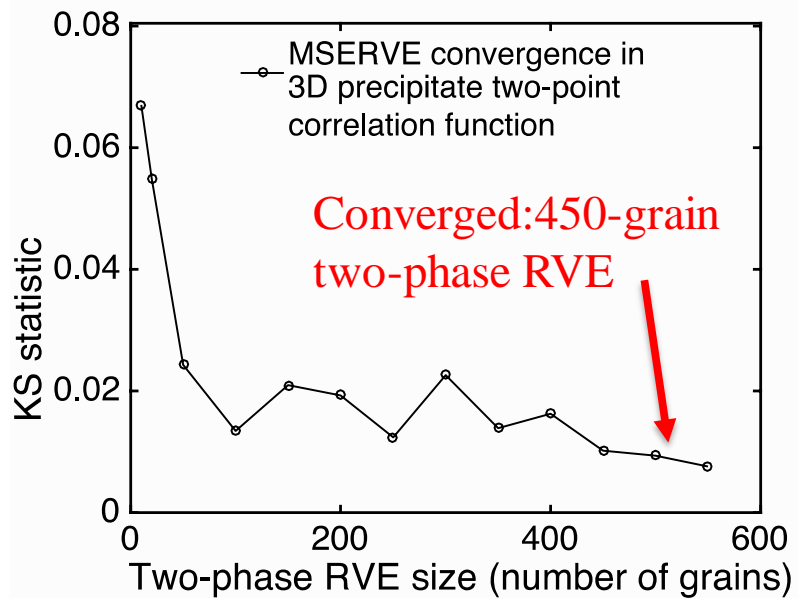
# Extending M-SERVE

## Al-7075 Alloys & Ni-based Superalloys

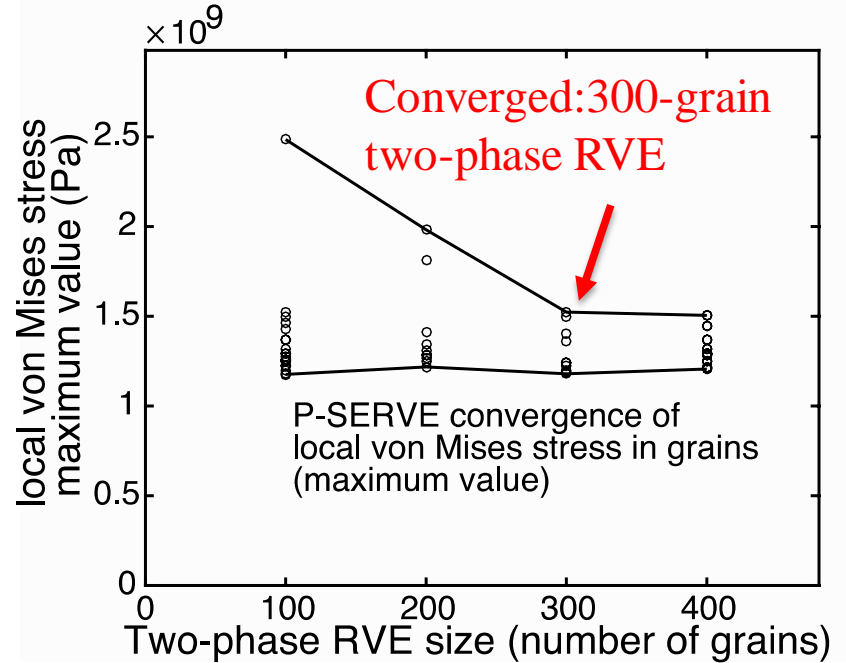


# Convergence Tests for M-SERVE and P-SERVE

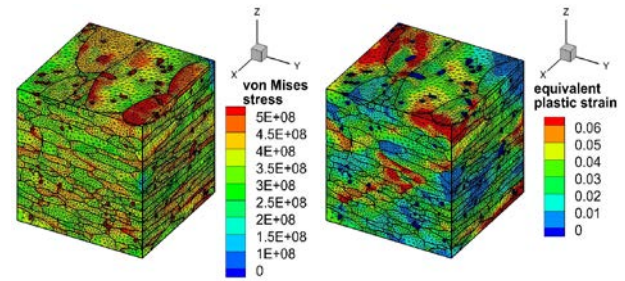
## Particle two-point correlation function



## Maximum Stress in Grains



Kolmogorov–Smirnov test (K–S test):  
Convergence tests with error metrics





# Statistically Equivalent Representative Volume Element (SERVE)

RVE-based methods of property determination are concerned only with the size of the representative microstructural domain.

- Affine transformation-based displacement boundary condition,
- Uniform traction boundary condition
- Periodic boundary condition

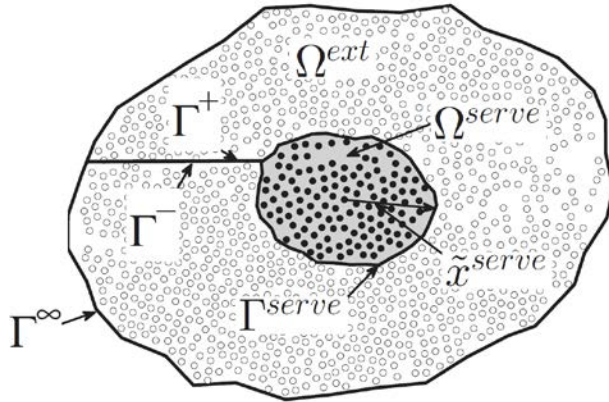
No consideration is given to the appropriateness of the boundary conditions applied to the RVE for solving the micromechanics.

## Results in significant over-estimation of the RVE

Optimally, the SERVE should:

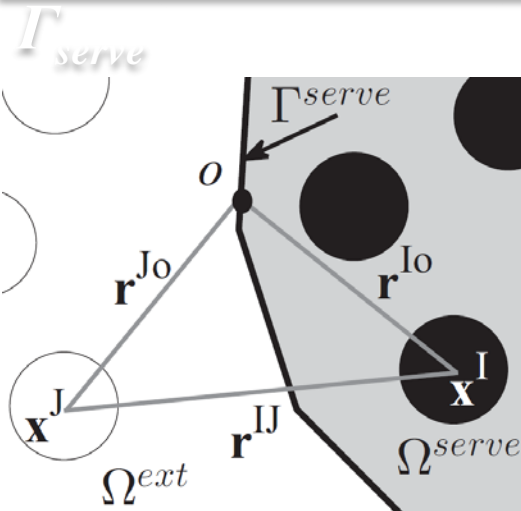
- Encompass the region required to represent essential mechanisms
- Represent boundary conditions that reflect the effect of the region exterior to the SERVE domain.

# Statistically Informed Green's Function (SIGF) Approach



Represent boundary conditions that reflect the effect of the region exterior to the SERVE domain.

## Perturbed Displacement at $\mathbf{x}$ in MVE and on SERVE Boundary

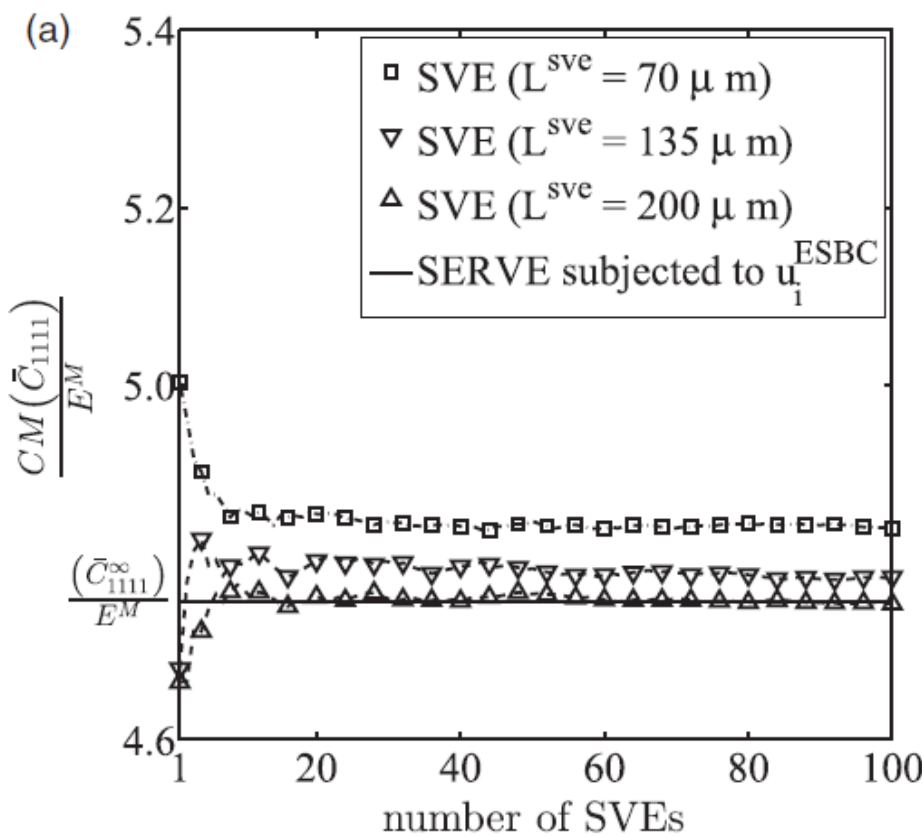
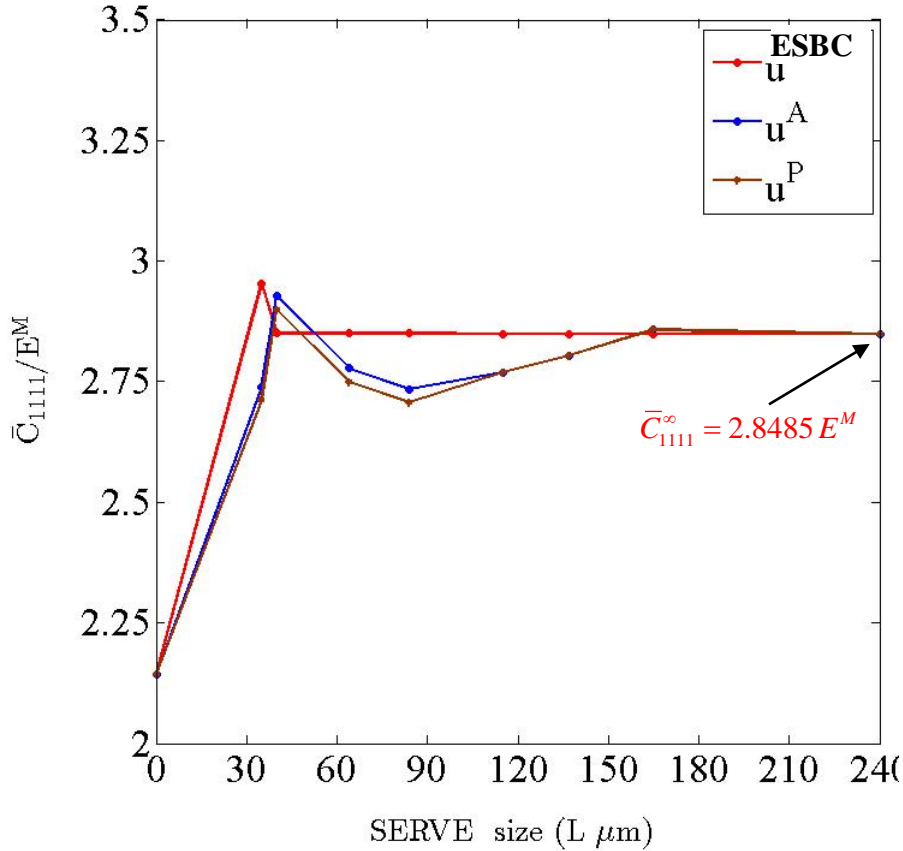


$$u_i^*(\mathbf{x}) = \left( \int_{\Omega^{mve} \setminus \Omega_F} S_2(\mathbf{r}') L_{imn}(\mathbf{r}') A_{mnkl}(\mathbf{r}') d\Omega \right) \epsilon_{kl}^M$$

$$A_{ijkl}(\mathbf{x}) = \left[ t^F(\mathbf{x})(S_{ijab} + M_{ijab}) - \int_{\Omega^{mve} \setminus \Omega_F} S_2(\mathbf{r}) \hat{G}_{ijmn}(\mathbf{r})(S_{mnpq} + M_{mnpq})^{-1} \hat{G}_{pqab}(\mathbf{r}) d\Omega \right]^{-1} \left[ (S_{abmn} + M_{abmn})^{-1} \int_{\Omega^{mve} \setminus \Omega_F} S_2(\mathbf{r}) \hat{G}_{mnkl}(\mathbf{r}) d\Omega - \frac{1}{2}(\delta_{ak}\delta_{bl} + \delta_{al}\delta_{bk}) \right] \epsilon$$

Ghosh and Kubair *JMPS*, (2016), Kubair, Ghosh *IJSS* (2017), Kubair, Pinz, Przybyla, Ghosh *J. Comp. Mat.* (2018)

# Homogenized Elastic Stiffness with $u^A$ or $u^P$ and $u^{ESBC}$ on $\Gamma^{serve}$



**SERVEs subjected to ESBCs converge for smaller volumes**

**Fast convergence wrt SVEs**

*Kubair, Pinz, Przybyla, Ghosh J. Comp. Mat. (2018)*

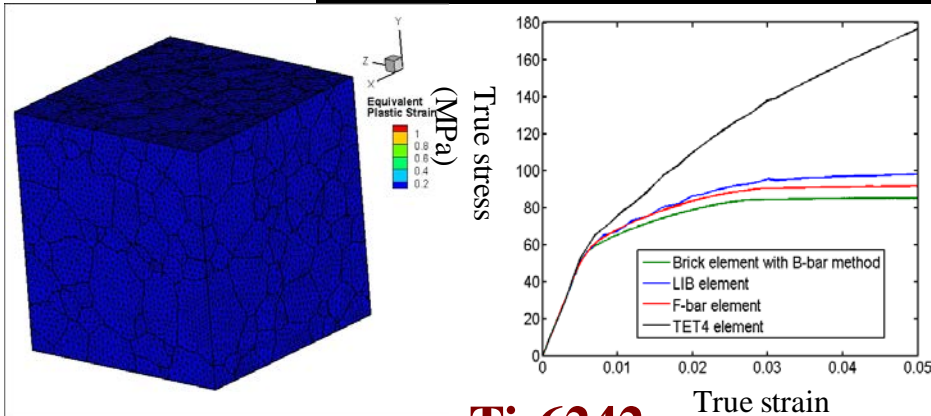
*Ghosh and Kubair JMPS, (2016), Kubair, Ghosh IJSS (2017)*



# Methods of Analysis and Simulation at Relevant Scales: Accuracy and Efficiency

- 1. Finite Element Methods: CPFEM:** *Bronkhorst, Kalidindi, Anand (1992). Kalidindi, Schoenfeld (2000), Bridier, McDowell, Villechaise, Mendez (2009), Mayeur, McDowell, Neu (2008), Busso, Meissonier, O'Dowd (2000), Dunne, Kiwanuka, Wilkinson (2012), Shahba, Ghosh (2016), Venkataramani, Ghosh, Mills (2007), Roters, Eisenlohr, Hantcherli, Tjahjantoa, Bieler, Raabe (2010)*
- 2. Boundary Element Methods**
- 3. Fast Fourier Transform Methods** *Moulinec, Suquet (1998), Lebensohn, Kanjarla, Eisenlohr (2012), Eisenlohr, Diehl, Lebensohn, Roters (2013)*
- 4. Phase Field Methods:** *Yamanaka, Tomohiro, Takaki, Tomita, Yoshino (2009)*
- 5. Peridynamics:** *Sun, Sundararaghavan (2014), Luo, Ramazani, Sundararaghavan (2018)*

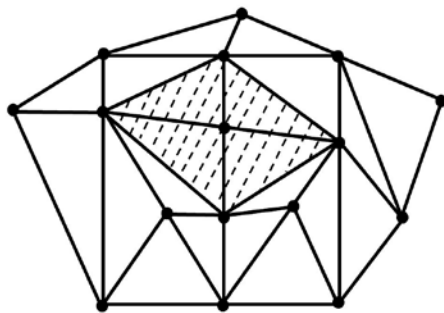
# Element Stabilization to Avoid Volumetric Locking in CPFEM



**Ti-6242**

Incompressibility enforced over a non-overlapping patch of elements

## F-bar Patch Method

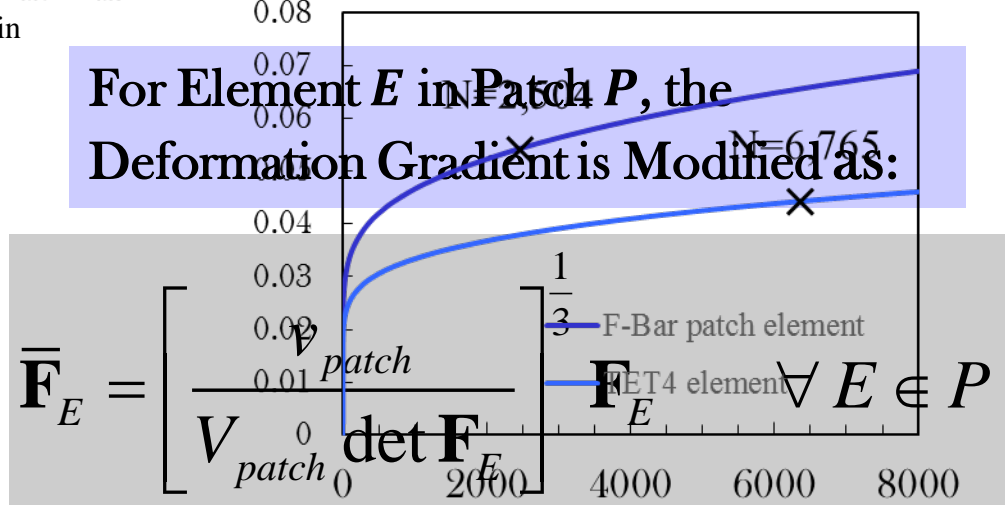


• Node — Edge of triangular element

$$\mathbf{F} = \mathbf{F}_{iso} \mathbf{F}_{vol}$$

$$\mathbf{F}_{iso} = (\det \mathbf{F})^{-\frac{1}{3}} \mathbf{F} = (\det \mathbf{F})^{\frac{1}{3}} \mathbf{I}$$

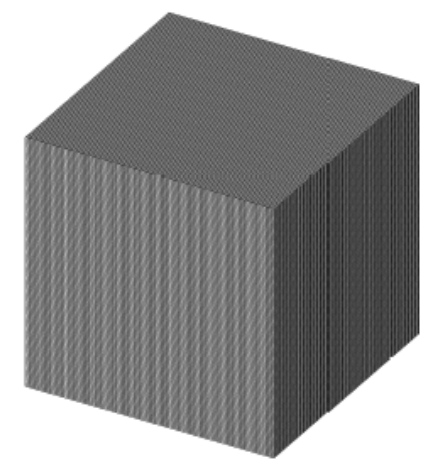
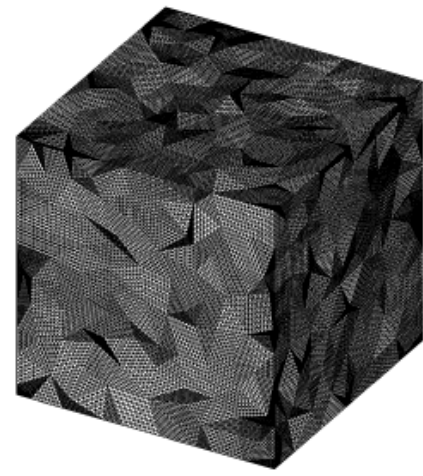
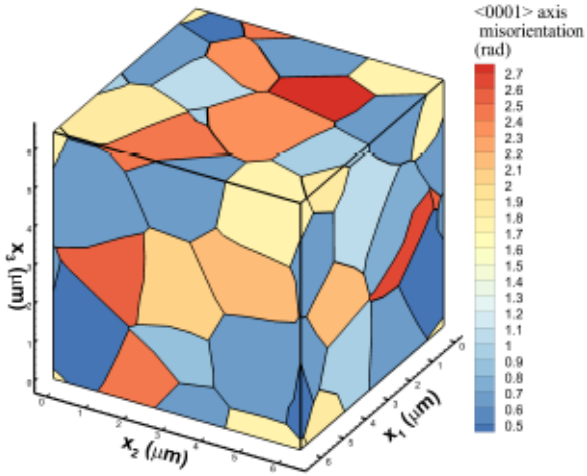
For Element  $E$  in Patch  $P$ , the Deformation Gradient is Modified as:



$$\bar{\mathbf{F}}_E = \begin{bmatrix} \frac{1}{3} \frac{v_{patch}}{V_{patch}} \det \mathbf{F}_E & & \\ & \mathbf{F}_E & \\ & & \mathbf{I} \end{bmatrix} \quad \forall E \in P$$

$v_{patch}$  : deformed volume of patch  
 $V_{patch}$  : reference volume of patch

# Accuracy and Efficiency of CPFEM

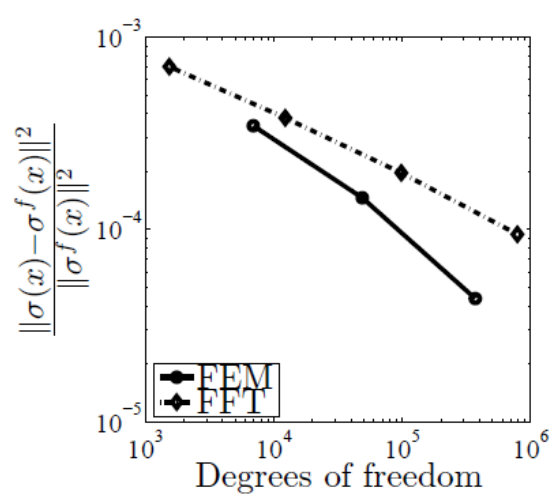
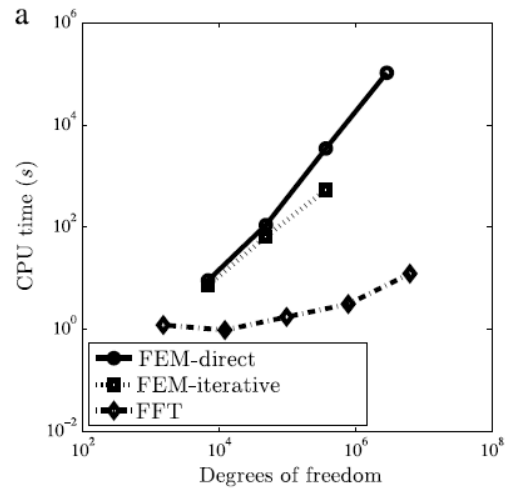


**25 grains, high misorientation, Periodic BC, uniaxial load**

**ABAQUS STANDARD**  
5640192 TET Elements

**FFT Code CraFT**  
128<sup>3</sup> Grid Points

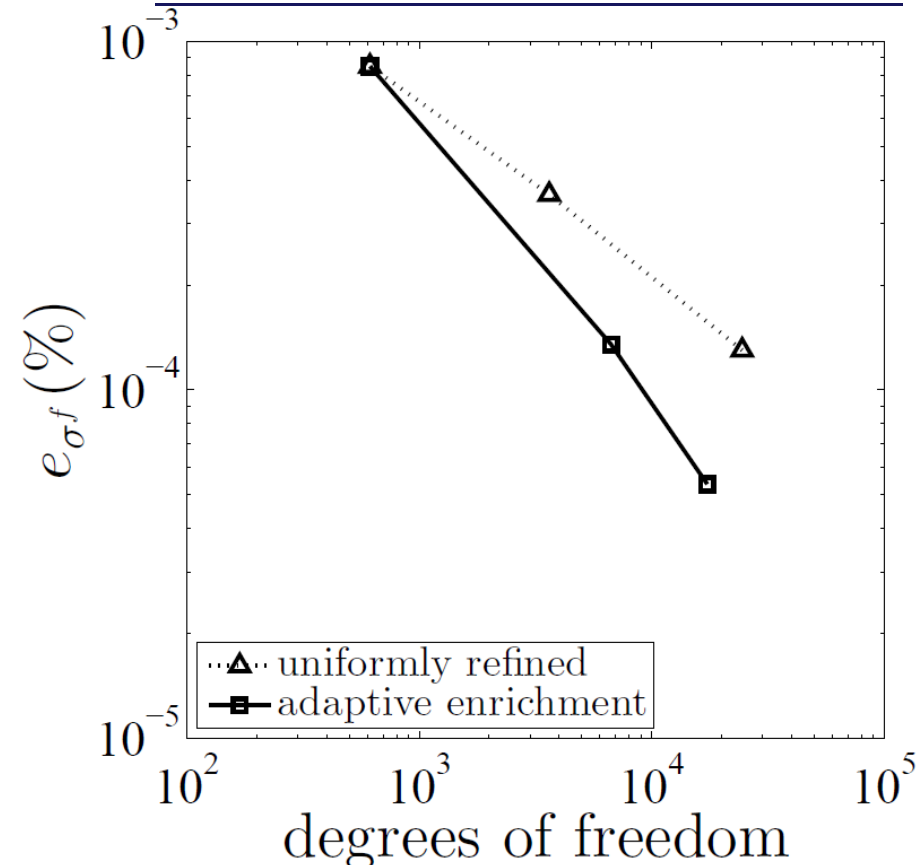
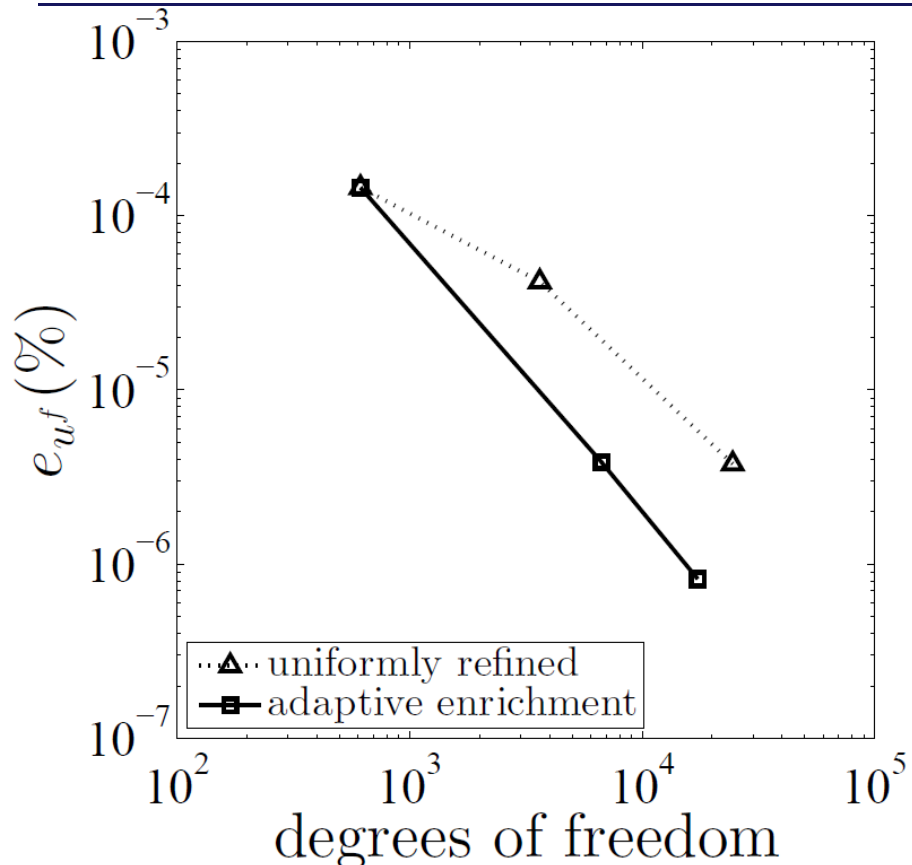
*(Moulinec and Suquet, 1994)*



While the speed of FFT could be orders of magnitude faster, the convergence rate of the global error could be slower in the presence of heterogeneities

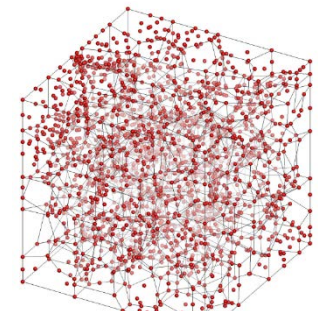
*Azdoud, Cheng, Ghosh, CMAME, (2017), Azdoud, Ghosh, CMAME, (2017).*

# A Computationally Efficient and Accurate CPFEM-based Hierarchical Method



**The adaptive method is  $\sim 20$  faster than uniformly refined CPFEM**

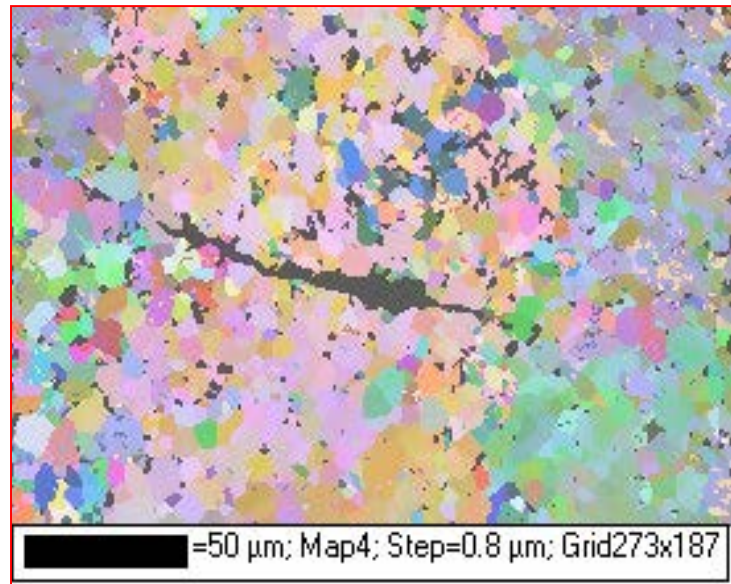
Y. Azdoud, S. Ghosh, *CMAME*, (2017)  
Y. Azdoud, J. Cheng, S. Ghosh, *CMAME*, 2017



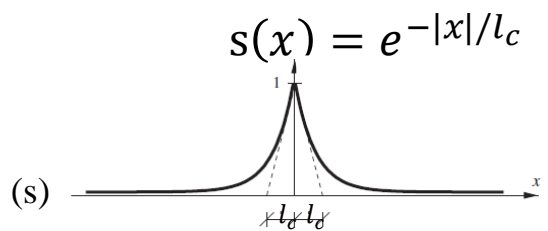
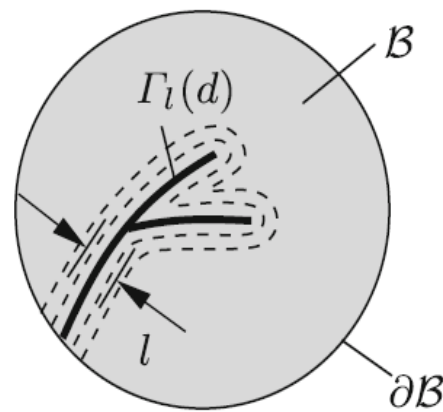
# Crack Propagation

## Coupling Phase-Field Modeling with CPFEM

Introduces a thermodynamic consistent plasticity-informed continuum formulation for crack propagation.



Continuous Auxiliary Global Field to Approximate Sharp Crack Discontinuities.



Diffused crack

Allen-Cahn equations

$s$  is the phase field variable (order parameter):  
 $s \in [0, 1]$ ;  $s = 0$  perfect solid;  $s = 1$  fully cracked

Sharp crack of area  $\Gamma$  is regularized by a crack surface density function  $\gamma_l(s, \nabla s)$  which measures a spatially regularized total crack surface  $\Gamma_l(s)$

$$\Gamma_l(s) = \int_v \gamma_l(s, \nabla s) dV$$

$$\gamma_l(s, \nabla s) = \frac{1}{2l} s^2 + \frac{l}{2} |\nabla s|^2$$

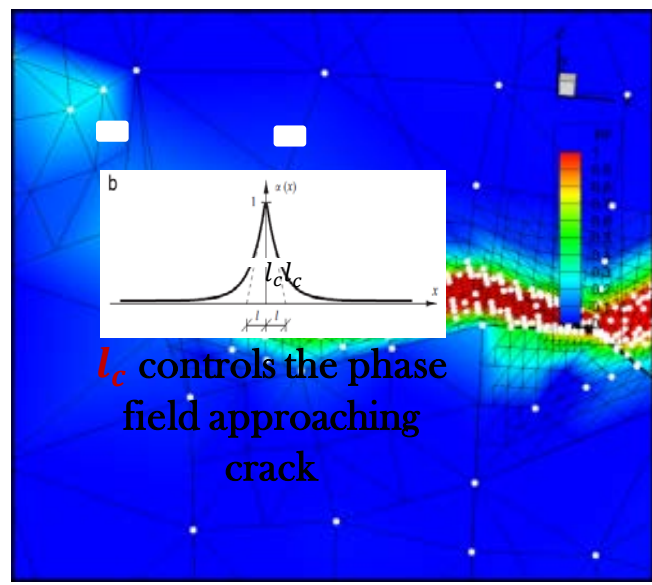
Miehe, Hofacker, Welschinger (2010)



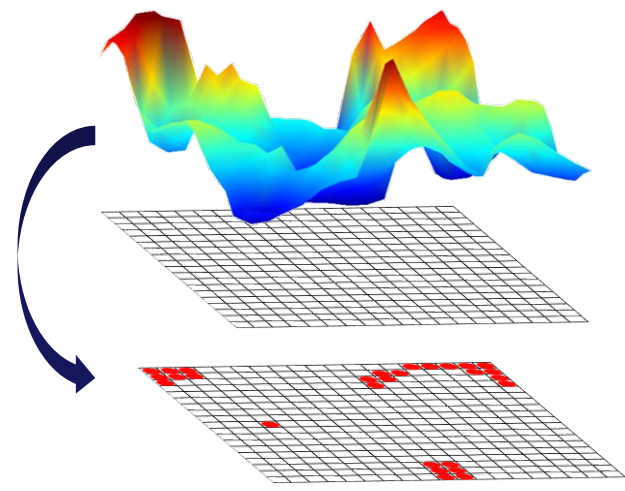
# Challenge with Phase Field Modeling Requires Extremely Fine Resolution

## Adaptive Projection to Phase-Field Model: Wavelet-Enriched Hierarchical FEM

- *Hierarchical FE* preserves coarse mesh and adds multi-resolution shape functions
- Wavelet based Hierarchical FEM allows for optimum computation of phase field



ets



Phase field

Coarse mesh

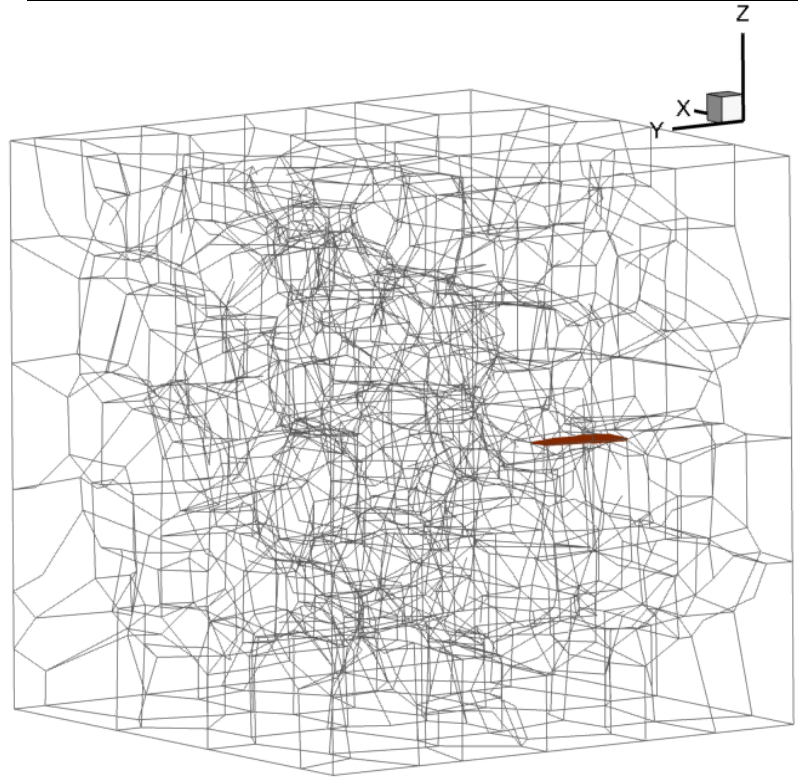
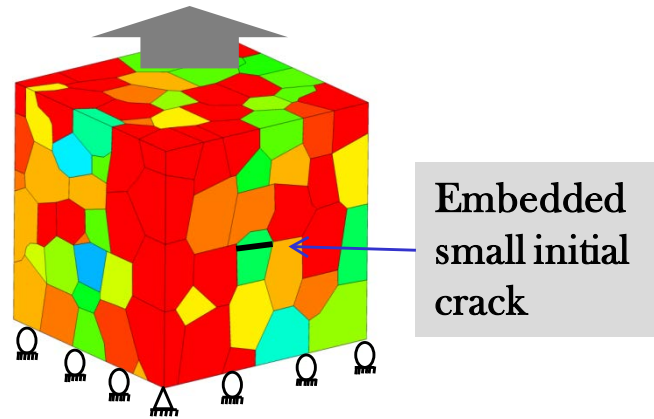
Enhancement

Hierarchical adaptive FEM  
With wavelet enhancement  
~10,000 nodes are required

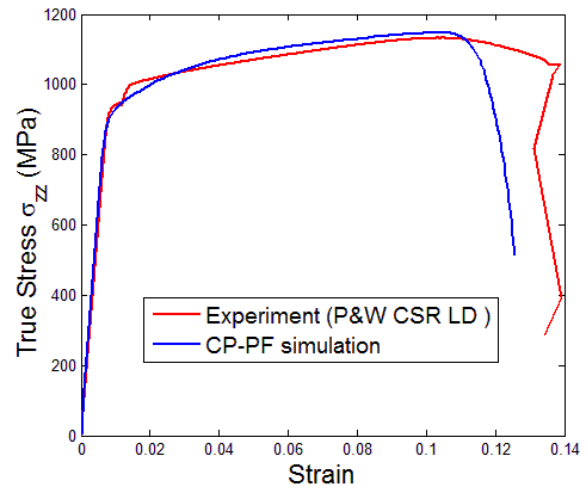
FEM with global fine mesh needs  
~1,000,000 nodes

# Crack Propagation in Ti-64 Microstructure using Wavelet Enriched Hierarchical FEM

## Visualization of crack propagation



## Macroscopic stress-strain



The same result would be obtained using conventional FEM with a fine mesh containing **~7,000,000** nodes

In the wavelet-enhanced adaptive hierarchical FEM, only **~300,000** nodes are required



# Accelerated Time Integration Methods and Multi-time Scaling

1. Time Acceleration for Fatigue Modeling
2. Subcycling for Modeling Sub-Domains with Disparate Time Requirements

# Wavelet Transformation Based Multi-Time Scaling Methodology (*WATMUS*)

## Wavelet Decomposition of Nodal Displacements

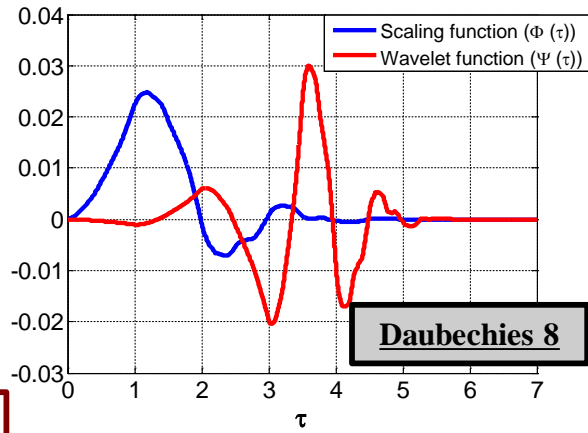
$$u(N, \tau) = \sum_k c_k(N) \psi_k(\tau)$$

Coefficients of wavelet basis

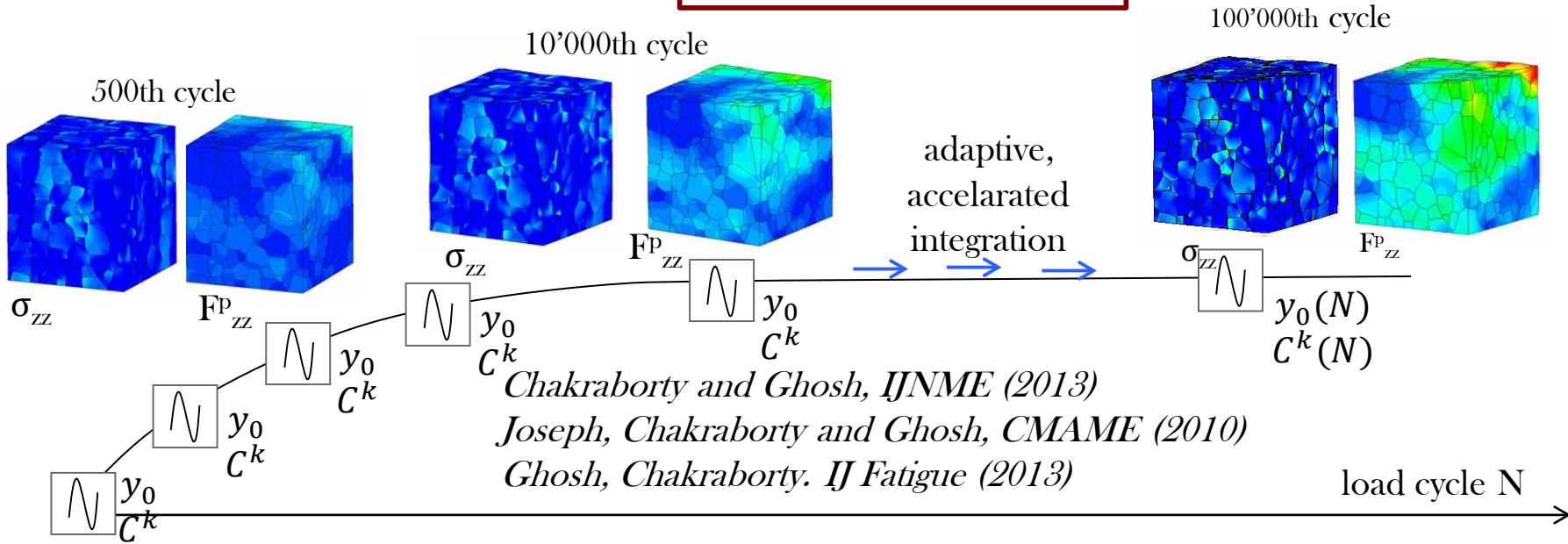
- Depends on cycle scale ( $N$ )
- *Independent of fine scale.*

Wavelet basis function

- Fine scale ( $\tau$ ) behavior
- *Independent of ( $N$ )*



$$\phi_{m,n} = 2^{\frac{m}{2}} \phi(2^m \tau - n)$$

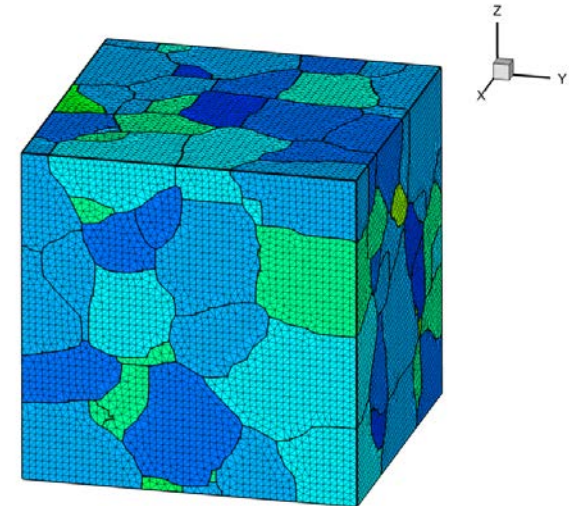


# Adaptive Subcycling in CPFEE Simulations for Twin Evolution

- Twin bands require very high resolution FE mesh
- Twin formation intensifies time steps with localization within twin bands

Strain localization within twin bands

97% elements can converge at  $\Delta t=10s$   
 3% elements can converge at  $\Delta t=0.03s$



1. Spatial domain decomposition

2. Solve separately using fine and coarse time steps

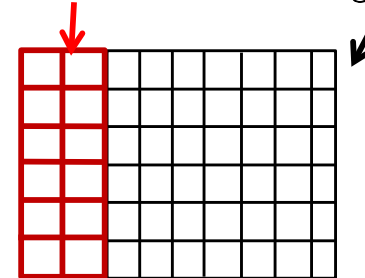
3. Couple and iterate

Over 1 order magnitude faster

Cheng, Ghosh, *JMPS*, (2017), Ghosh,  
 Cheng, Ghosh, *CM*, (2017)

Deformation localized region (twin)

Low deformation rate region



$$\Delta \mathbf{u} = \left( \tilde{\mathbf{K}}_{t+\Delta t}^F + \mathbf{K}_{t+\Delta t}^C \right)^{-1} \left\{ \mathbf{f}_{t+\Delta t}^{ext} - \tilde{\mathbf{f}}_{t+\Delta t}^{int,F} - \mathbf{f}_{t+\Delta t}^{int,C} \right\}$$

$$\Delta \mathbf{u}_{t \rightarrow t+\Delta t}^{corrector} = \Delta \mathbf{u}_{t \rightarrow t+\Delta t}^{trial} + \Delta \tilde{\mathbf{u}}^F + \Delta \mathbf{u}$$



# Hierarchical Upscaling: Homogenization Models from Micromechanical Simulations

Microscale

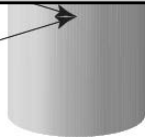
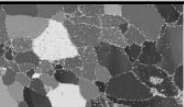
Macroscale

effective response



- ❑ **Variational framework:** Ponte Castaneda (1991,1992), Hughes (1998)
- ❑ **Self-consistent method:** Kroner (1958), Budiansky and Wu (1962), Lebensohn and Tom´e (1993)

- ❑ Many of these models are not suitable for representing complex material behavior e.g., anisotropy, path-dependent behavior etc..
- ❑ Could be computationally prohibitive for large-scale structural simulations.

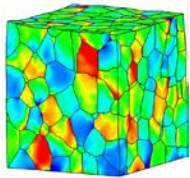


- ❑ **Reduced order models:** Fish, et. al. (2012), Ghosh, et. al (2007, 2009)
- ❑ **Distribution-Enhanced Homogenization Framework:** Alleman, Ghosh, Luscher, Bronkhorst, JMPS, 2015

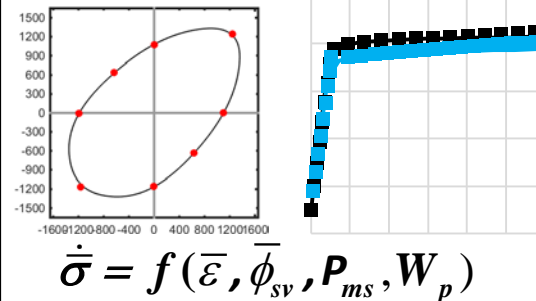
# A Promising Alternative

## Parametrically Homogenized Constitutive Models (PHCM)

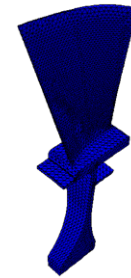
Detailed  
Micromechanical  
Model



PHCM



Simulations with PHCM



- ❑ Thermodynamically consistent, continuum scale, reduced order models with parametric delineation of morphological distributions
  - ❑ Constitutive parameters and their evolutions expressed in terms of parameters of statistical distribution functions
  - ❑ *Machine Learning* for function forms of coefficients
- ❑ Easily incorporated in commercial codes through UMAT
  - ❑ Significant efficiency without loss of accuracy from physics-based models
  - ❑ Direct connection to the “materials by design”

## Elastic-Plastic Constitutive Equations for Macro-modeling

### Elasticity

$$\mathbf{S} = \mathbb{C}^{PHCM}(g_1, g_2, g_3) : \mathbf{E}$$

$$\mathbb{C}^{PHCM} = g_1 \mathbb{C}_{(x)}^{single} + g_2 \mathbb{C}_{(y)}^{single} + g_3 \mathbb{C}_{(z)}^{single}$$

### Anisotropic Plasticity

$$\mathbf{D}^p = \dot{\gamma}_0 (Y/Y_0)^{1/m} \mathbf{N}$$

$$Y = [ (|\Sigma_1| - k\Sigma_1)^a + (|\Sigma_2| - k\Sigma_2)^a + (|\Sigma_3| - k\Sigma_3)^a ]^{1/a}$$

Transformed stress  $\Sigma = \mathbb{L}(\overline{OMA}_{ij}^\mu) : \mathbf{S}$

Anisotropy Tensor

### Hybrid Voce Type Hardening and Grain Size Dependency

$$Y(\varepsilon_p) = Y_0(\bar{D}_\mu, \bar{D}_\sigma) + \alpha(\bar{A}_{\theta mis}) \left( 1 - \exp \left[ - \left( \frac{\varepsilon_p}{\bar{\varepsilon}_p(\overline{OMA}_{ij}^\mu)} \right)^{0.75} \right] \right) + h_0(\overline{OMA}_{ij}^\mu, \bar{D}_\mu) \varepsilon_p$$

Initial Yield Strength Hardening Modulus Misorientation, grain size distn.

## Microstructural Parameters inputs to the UMAT from EBSD scans

$g_1, g_2, g_3$   
Texture intensity parameters  
 $v_1, v_2, v_3$   
Material symmetry axes

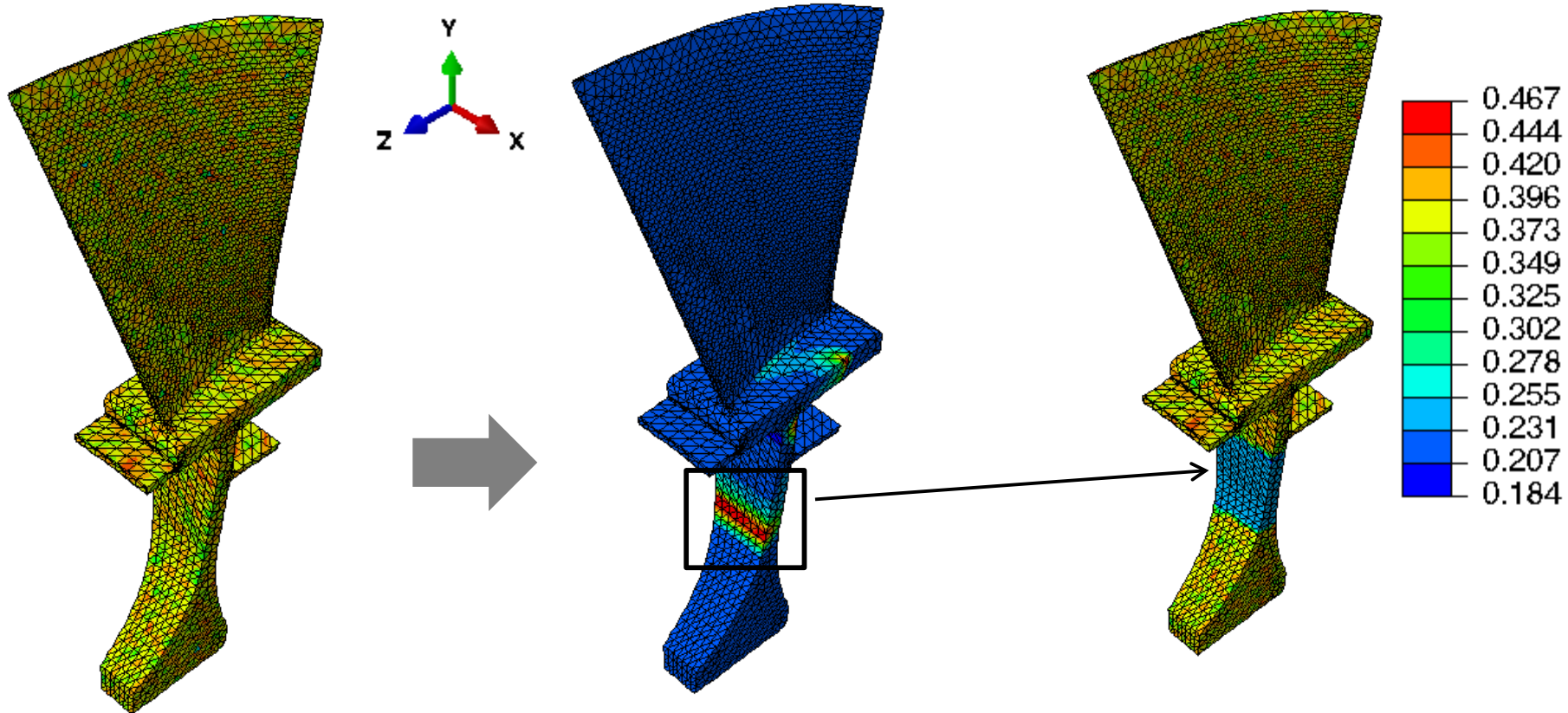
$\overline{OMA}_{ij}^\mu$   
Crystallographic  
Orientation Measure  
in Material Frame

$\overline{OMA}_{ij}^\mu$   
 $\bar{A}_{\theta mis}$   
 $\bar{D}_\mu, \bar{D}_\sigma$





**Design Objective:** Change the material properties in the localized band to reduce the plastic strains



Original  $\overline{OMA}_{33}$   
from the EBSD

Original properties leading  
to localized plastic strain

Modified  $\overline{OMA}_{33}$  in  
the localized region

With unraveling of new mechanisms and defect structures in complex heterogeneous materials and structures:

- ❑ Using conventional tools are in most cases insufficient
- ❑ Need for novel methods and algorithms in Computational Mechanics and Computational Materials Science is becoming more meaningful.
- ❑ Better methods of reaching large audiences with these tools